Question 1

Marks

(a) Prove that $\frac{d}{dx}(\frac{1}{2}v^2) = \ddot{x}$. **2**

(b) The acceleration of a creature is given by $\ddot{x} = -\frac{1}{2}u^2e^{-x}$ where x is the displacement from the origin and u is the initial velocity at the origin.

Given that u = 2 and v is the velocity at time t.

(i) Show that $v^2 = 4e^{-x}$. **2**

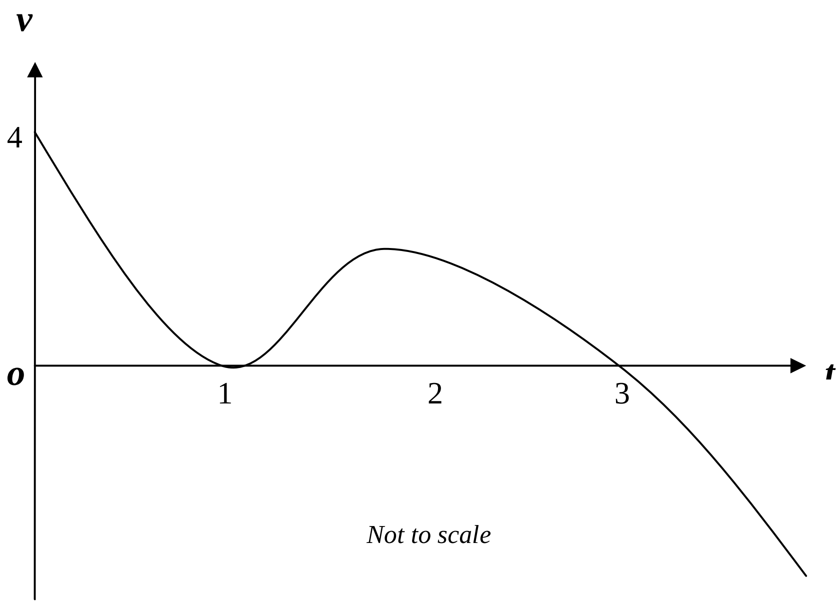
(ii) Explain why $v > 0$ for $t \geq 0$. **1**

(iii) Find x in terms of t. **2**

(iv) Describe the motion of the creature (Give reasons) **2**

Question 2 (Start a new Page)

(a) A particle P moves along a straight line. A velocity-time graph for P is shown below. The graph is tangent to the x axis at x =1.



(i) Between what times does the particle travel to the right ? **1**

(ii) Sketch a displacement – time graph for P given that the particle starts 2 units to the left of the ***O*****.** **2**

Question 2 continued **Marks**

(b) When a person dies, the temperature of their body will gradually decrease from 37 $^\circ$C(normal body temperature), to the temperature of the surroundings. Newton's law of cooling states that the temperature of the cooling body changes at a rate proportional to the difference between the temperature of the body and the temperature of its surroundings.

That is $\dfrac{d\theta}{dt} = -K(\theta - R)$ ....(1)

Where K is a positive constant, θ is the temperature of the body after t hours, and R is the temperature of the surroundings.

A person was found murdered in his house. Police arrived on the scene at 10:56 pm. The temperature of the body at that time was 31 $^\circ$C, and one hour later it was 30 $^\circ$C. The temperature R of the room in which the body was found was 22 $^\circ$C.

(i) Show that $\theta = 22 + Ae^{-Kt}$ is a solution of equation (1), where A is a constant. **1**

(ii) Find the exact values of A and K. **2**

(iii) Determine the time when the murder was committed, correct to the nearest minute. **3**

Question 3 (Start a new Page)

(a) From the letters of the word RENEGADE, three are taken at random and placed in a line.

(i) How many different 3 letter sequences are there with exactly one E in the sequence? **1**

(ii) How many different 3 letter sequences are there altogether? **3**

(b) The speed , v cm/s, of a particle moving along the x-axis is given by $v^2 = 72 - 12x - 4x^2$.

(i) Show that the motion is simple harmonic. **2**

(ii) Find the period and amplitude of the motion. **3**

Question 4 (Start a new Page) **Marks**

(a) Katie is a member of a 9-player softball team.

(i) In how many ways can they bat if Katie bats in the 9^{th} position? **1**

(ii) There are two left-handers in the team. If the batting order is randomly selected, what is the probability that the left-handers will be in the 1^{st} and 9^{th} positions? **2**

(b) The rate of change of the number of rabbits infected by a disease is given by the equation $\frac{dN}{dt} = N(100 - N)$, where N is the number of infected rabbits at time t years. There are 100 rabbits originally.

(i) If k is a constant, show that $N = \frac{100}{1+ke^{-100t}}$ satisfies the above equation **2**

.

(ii) If at time t = 0 one rabbit was infected, after how many days will half the number of rabbits be infected, correct to two decimal places? **2**

(iii) Sketch the graph of $N = \frac{100}{1+ke^{-100t}}$. **2**

Question 5 (Start a new Page)

(a) In March this year, the 8 quarterfinalists of the 2008 Champions League Football competition were randomly drawn into 4 quarterfinals .

4 of the quarterfinalists were English teams:
Manchester United, Liverpool, Chelsea and Arsenal.

The other 4 quarterfinalists were from mainland Europe:
Rooma, Barcelona, Schalke and Fenerbahce.

Note that this is a knock-out competition where the team beaten will be out of the competition..

(i) Find the number of different quarterfinal draws possible. **1**

(ii) What was the probability that at least one quarterfinal was played between 2 English teams? **2**

Question 5 continued

Marks

(b) The depth x metres of the water in a certain South Coast harbour is found to vary approximately according to the equation $\ddot{x} = -\frac{x}{4}$.

Given that t is the time in hours and it is known that the difference between high and low tide is 4 metres.

(i) Prove that the time between successive high tides is 12.6 hours, correct to the nearest $\frac{1}{10}$ of an hour. **2**

(ii) Find the rise in the water level during the first hour after low tide. Give your answer in metres, correct to two decimal places. **2**

(iii) Find the rate at which the level is falling two hours after high tide. Give your answer in metres per hour, correct to two decimal places. **2**

Question 6 (Start a new Page)

(a) A particle moves such that its displacement (x) is given by the equation: $x = 3\cos 5t + 4\sin 5t$, where t is the time taken. Find the maximum displacement of the particle. **2**

(b) Pete and Graham are both standing 50 metres apart on level ground. Pete throws a ball from a height of 1.9metres which Graham catches 2 seconds later (without bouncing), also at a height of 1.9 metres.

You may assume:

1. there is no air resistance and the value of g is 10m/s^2
2. the equations of motion are :

$$\dot{x} = V\cos\alpha \qquad \dot{y} = -10t + V\sin\alpha$$
$$x = Vt\cos\alpha \qquad y = -5t^2 + Vt\sin\alpha + 1.9$$

where V is the initial velocity , α is the angle of projection, t is the time taken and the origin is at Pete's feet.

(i) Find the initial speed and the angle of projection to the nearest minute. **3**

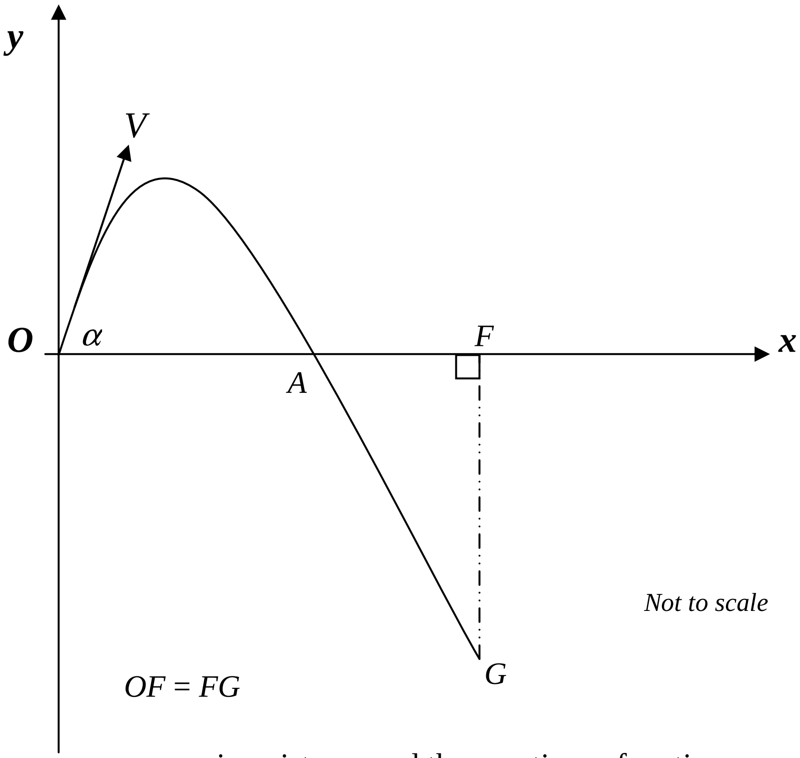
(ii) Find the maximum height of the ball above the ground. **2**

(iii) Pete throws another ball with the same initial velocity and from the same starting height (1.9 metres above the level ground), but he wants to maximize the distance he throws horizontally. **3**

How far away should Graham now stand away from Pete in order to catch this second throw without the ball bouncing and at a height of 1.9 metres?

Question 7 (Start a new page) **Marks**

In the diagram above, a projectile is fired from a point ***O*** at the top of a vertical cliff. Its initial speed is V m/s and its angle of projection is α. Let the acceleration due to gravity be g m/s^2.



You may assume no air resistance and the equations of motion are:

$$\dot{x} = V\cos\alpha \qquad \dot{y} = -gt + V\sin\alpha$$

$$x = Vt\cos\alpha \qquad y = -\frac{1}{2}gt^2 + Vt\sin\alpha$$

Let G be the point on the projectile's path where the distance below the origin equals the distance to the right of the origin. That is, $OF = FG$ on the diagram above.

(i) Prove that the time taken for the projectile to reach G is **2**

$$\frac{2V(\sin\alpha + \cos\alpha)}{g} \text{ seconds.}$$

(ii) Hence, show that $OF = \frac{V^2}{g}(\sin 2\alpha + \cos 2\alpha + 1)$ metres **2**

(iii) Let A be the point on the projectile's path where it is level with the point of projection. If $OF = \frac{4}{3}OA$, find α, correct to the nearest degree. **4**

END

Y12 MATHS EXT 1 TERM 2 ASSESSMENT TASK 3 2010

MATHEMATICS Extension 1 : Question 1

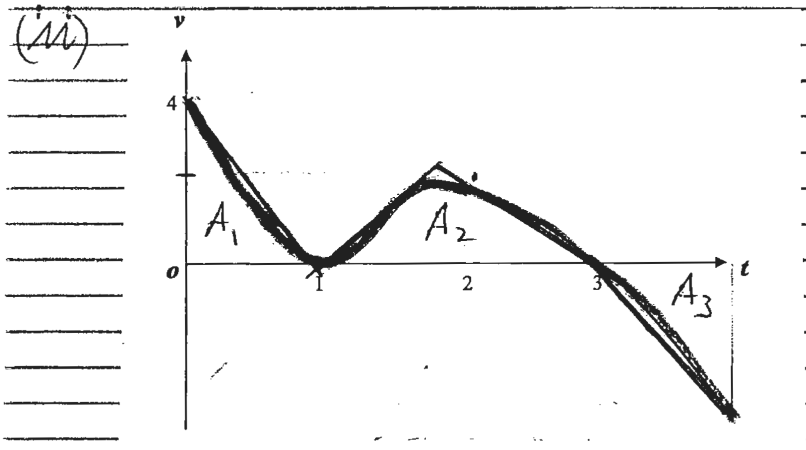
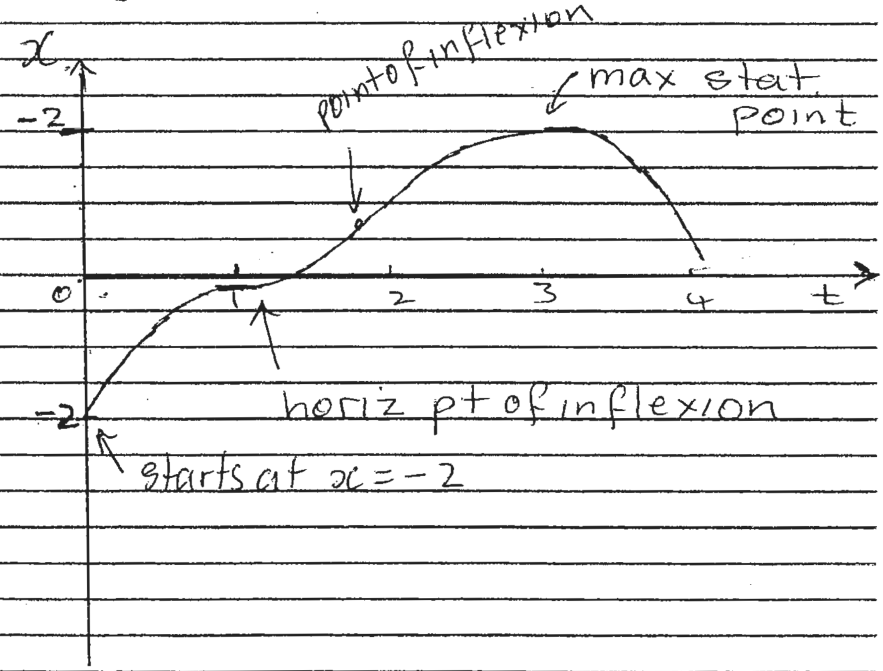
Suggested Solutions	Marks	Marker's Comments
(a) LHS: $\frac{d}{dx}(\frac{1}{2}v^2) = \frac{d}{dv}(\frac{1}{2}v^2).\frac{dv}{dx}$ $\frac{1}{2}$ $= v.\frac{dv}{dx}$ $\frac{1}{2}$ $= \frac{dx}{dt}.\frac{dv}{dx}$ $\frac{1}{2}$ $= \frac{dv}{dt}$ $\frac{1}{2}$ $= \ddot{x}$ RHS RHS $\ddot{x} = \frac{dv}{dt}$ $= \frac{dv}{dx}.\frac{dx}{dt}$ $= \frac{dv}{dx}.v$ $= \frac{dv}{dx}.\frac{d(\frac{1}{2}v^2)}{dv}$ $= \frac{d}{dx}(\frac{1}{2}v^2)$ LHS	$\frac{1}{2}$ $\frac{1}{2}$ 1	$\boxed{2}$
(b) (i) Data: $t=0$ $x=0$ $v=u=2$ $\ddot{x}=-2$ $\rightarrow 2u/s$ $\oplus$ x, 0 $\ddot{x} = -\frac{1}{2}u^2e^{-x} = -2e^{-x}$ so $\frac{d}{dx}(\frac{1}{2}v^2) = -2e^{-x}$ $\therefore \frac{1}{2}v^2 = \int -2e^{-x}dx$ $\frac{1}{2}v^2 = +2e^{-x}+c$ 1 when $t=0$ $x=0$ $v=2$ so $2 = 2e^0 + c$ $c=0$ 1 $\therefore \frac{1}{2}v^2 = 2e^{-x}$ so $v^2 = 4e^{-x}$ qed $\int_{u=2}^{v} d(\frac{1}{2}v^2) = \int_0^x -2e^{-x}dx$ $\frac{1}{2}v^2\Big]_2^v = +2e^{-x}\Big]_0^x$ $\frac{1}{2}$ $\frac{1}{2}[v^2-4] = 2[e^{-x}-e^0]$ $\frac{1}{2}$ $v^2-4 = 4(e^{-x}-1)$ 1 $\therefore v^2 = 4e^{-x}$ qed.		$\boxed{2}$
(ii) Since $v^2 = 4e^{-x} > 0$ as $e^{-x} > 0$ $\forall x \in \mathbb{R}$ $\therefore v^2 > 0$ $\therefore v^2 \neq 0$ so $v \neq 0$ $\Rightarrow$ $v<0$ or $v>0$!!! $\therefore$ never stops let alone change direction But as $t=0$ $v=2>0$ $\therefore$ moves to the right initially $\therefore$ $v>0$ only for $t \geq 0$.	$\frac{1}{2}$ $\frac{1}{2}$	if $v^2>0$ only $v<0$ or $v>0$ why NO why Yes $x = 2\ln(t+1)$ $v = \frac{2}{t+1} > 0$ for $t \geq 0$ $\boxed{1}$
(iii) $v = \sqrt{4e^{-x}} = 2e^{-\frac{1}{2}x}$ from (ii) $v>0$ $\frac{dx}{dt} = 2e^{-\frac{1}{2}x}$ $\int e^{\frac{1}{2}x}dx = \int 2dt$ $\frac{2}{1}e^{\frac{1}{2}x} = 2t+c$ $\frac{1}{2}$ $t=0$ $x=0$ $\therefore 2 = 0+c$ $\therefore c=2$ $\frac{1}{2}$ so $2e^{\frac{1}{2}x} = 2t+2$ $e^{\frac{1}{2}x} = t+1$ $\frac{1}{2}x = \ln(t+1)$ $\therefore x = f(t) = 2\ln(t+1)$ 1 $\int_0^x e^{\frac{1}{2}x}dx = \int_0^t 2dt$ $2e^{\frac{1}{2}x}\Big]_0^x = 2t\Big]_0^t$ $2[e^{\frac{1}{2}x}-e^0] = 2t$ $e^{\frac{1}{2}x}-1 = t$ $e^{\frac{1}{2}x} = t+1$ $\frac{1}{2}x = \ln(t+1)$ $\therefore x = 2\ln(t+1)$	$\frac{1}{2}$ $\frac{1}{2}$ 1	$\boxed{2}$

MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments
(iv) Note $\ddot{x} = -2e^{-x}$, $v = 2e^{-\frac{1}{2}x}$, $x = 2\ln(t+1)$		
at $t=0$: the creature is at the origin moving	$\frac{1}{2}$	
Initially to the right with a velocity of 2 u/s	$\frac{1}{2}$	
with an acceleration of -2 u/s^2		
For $t>0$: As the applied force ($\ddot{x} = -2e^{-x} < 0$)	$\frac{1}{2}$	
is negative (acting to the left) ∴		
the creature is being slowed down		
It continues to move to the right	$\frac{1}{2}$	
indefinitely but slows down but		$\boxed{2}$
it will never come to rest ($v>0$, $\ddot{x}<0$ (ii))		

\\CALLISTO\StaffHome$\WOH\JRAH M Fac Admin\Assessment info\Suggested Mk solns template_V4_half Ls.doc

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
2(a)(i) Particle moves to the right $0 \le t < 1$, $1 < t < 3$.	1	½ $0 \le t \le 3$ or similar ½ $t \ne 1$ no marks lost for including $t = 3$.
(ii)  $A_1 \approx 2$ units $A_2 \approx 2$ units (less than 3 units) $A_3 \approx 2$ units	2	
		1 shape. 1 correct values. stationary points had to be horizontal.

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
2(b)(i) $\theta = 22 + Ae^{-kt}$	1	
$\frac{d\theta}{dt} = -k(\theta - R) \quad R = 22$		
LHS $= \frac{d\theta}{dt}$		
$= -kAe^{-kt}$		
RHS $= -k(\theta - 22)$		
$= -k(22 + Ae^{-kt} - 22)$		
$= -kAe^{-kt}$		
LHS = RHS		1 full proof
$\therefore \theta = 22 + Ae^{-kt}$ is the general solution		
(ii) Let $t = 0$ be 10:56	2	
$t = 0 \quad \theta = 31$		
$\therefore 31 = 22 + Ae^{-k\times 0}$		
$\therefore 31 = 22 + A$		
$A = 9$		
when $t = 1 \quad \theta = 30$		1 $A = 9$
$30 = 22 + 9e^{-k\times 1}$		($A = 15$ if $t = 0$ when $\theta = 37$)
$\frac{8}{9} = e^{-k}$		
$k = -\ln(8/9) = \ln 9/8$		1 $k = \ln 9/8$
(iii) when $\theta = 37$	3	
$37 = 22 + 9e^{-kt}$		1 correct substitution
$\frac{15}{9} = e^{-kt}$		
$t = \frac{-1}{k}\ln\frac{15}{9}$		
$t = -4.337005078$		1 t value
Time interval = 4 HRS 20 MINS (nearest min)		
TIME $= 10{:}56 - 4{:}20$		
$= 6{:}36$ pm		1 answer

2010 Year 12 Term 3 Assessment – Ext 1 – QUESTION 3 (Marked by L.Kim)

Marking Scheme

(a) **Total: 4 marks**

RENEGADE has 3 E's and 5 other letters   EEE RNGDA

(i) 3 letter sequence with 1 E   _ _ _

→ 1 way for "E"

→ $^5C_2$ ways picking the other 2 letters

→ 3! To arrange the 3 letters in the line.

$\therefore$ **ANSWER = $^5C_2 \times 3! = 60$**   [1 Mark]

Alternatively→ 1 way to place the "E" and 5×4 ways to place the other 2 letters in a line, BUT there are 3 ways to place the "E" $\therefore$ ANSWER = $5 \times 4 \times 3 = 60$

(ii) Any 3 letter sequence

→ including 1 "E" = 60 from above

→ including 2 "E's" = $^5C_1 \times 3 = 15$ [1 Mark]

→ including 3 "E's" = 1 way only   [½ Mark]

→ sequence with no "E's" = $^5C_3 \times 3!$ [1 Mark]

Total = 60 + 60 + 15 + 1 = 136   [½ Mark]

*This question was quite poorly done, with students getting confused with the concepts of Permutations and Combinations

(b) **Total 5 Marks**

(i) $v^2 = 72 - 12x + 4x^2$ → $\ddot{x} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right) = -6 - 4x$ → [1 Mark]

Now $\ddot{x} = -4\left(x - \left(-\dfrac{3}{2}\right)\right)$ which is of the form $\ddot{x} = -n^2(x-b)$

$\therefore$ the particle obeys SHM about $x = b = -1\frac{1}{2}$, where $n = 2$

where $n = 2$ and $b = -\dfrac{3}{2}$.   → [1 Mark]

- If the used $v^2 = -n^2(a^2 - (x-b)^2)$ → 0 Marks
- If students had $\ddot{x} = -4\left(x + \dfrac{3}{2}\right)$ and had $\ddot{x} = -n^2(x-b)$ and *stated* $b = -1.5$, then full marks, HOWEVER, if didn't state value of b then lost ½ mark.
- If students had $\ddot{x} = -4\left(x + \dfrac{3}{2}\right)$ and $\ddot{x} = -n^2(x+b)$ then lost 1 mark

(ii) Period = $\dfrac{2\pi}{2} = \pi$ seconds → [1 Mark]

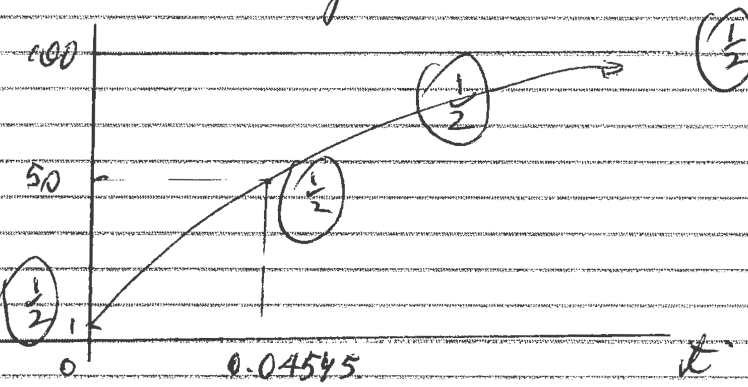
$72 - 12x + x^2 = 0$ to find extremity of motion.

$\therefore (x+6)(x-3) = 0$ → $\therefore x = 3$ or -6

Thus the motion oscillates between $-6 \le x \le 3$ → [1 Mark]

→ amplitude = ½ $(3 - -6) = \dfrac{9}{2}$ or 4.5 cm → [1 Mark]

MATHEMATICS Extension 1 : Question 4 T2 2010.

Suggested Solutions	Marks	Marker's Comments
a(i) $8! = 40320$	1	
(ii) $P(\;) = \dfrac{2 \times 7!}{9!} = \dfrac{1}{36}$	1 1	Some had sample space wrong
b(i) $N = \dfrac{100}{1+ke^{-100t}}$		
LHS $= \dfrac{dN}{dt} = \dfrac{-100\cdot(-100ke^{-kt})}{(1+ke^{-100t})^2} = \dfrac{10000ke^{-kt}}{(1+ke^{-100t})^2}$	1	
RHS $= N(100-N)$		
$= \dfrac{100}{1+ke^{-100t}} \cdot \left[100 - \dfrac{100}{1+ke^{-100t}}\right]$		
$= \dfrac{100\times100}{1+ke^{-100t}} \cdot \dfrac{1+ke^{-100t}-1}{1+ke^{-100t}}$		
$= \dfrac{10000ke^{-100t}}{(1+ke^{-100t})^2}$	1	
$\therefore$ LHS = RHS		
$t=0 \quad N=1 \rightarrow k=99$	1	
$50 = \dfrac{100}{1+99e^{-100t}}$		
$t = \dfrac{\ln 99}{100}$	$\frac{1}{2}$	
$= 0.04595$ yrs Time $= 16.77$ days	$\frac{1}{2}$	students assumed 365 days/yr.
"ii"  	2	t was defined in years. Scaling by many students poor. Many forgot locking point

<table>
<tr><th>Suggested Solutions</th><th>Marks</th><th>Marker's Comments</th></tr>
<tr>
<td>a) i) Question should read "Find the number of different quarter finals draws are possible"

MU can be picked against 7 sides
(leaving 6 teams still to fix up)
Next can pick opposition 5 ways
(leaving 4 teams)
Next can pick opposition 3 ways.
∴ Total draws $7 \times 5 \times 3 \times 1 = \underline{\underline{105}}$

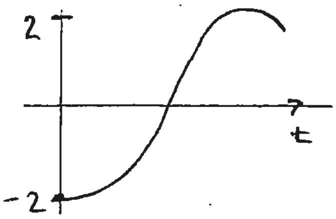
ii) Find ways that <u>no</u> English team plays each other.
MU can play any of 4 teams
L " " " " 3 "
C " " " " 2 "
A must play remaining team.
∴ $4 \times 3 \times 2 \times 1 = 24$ ways.

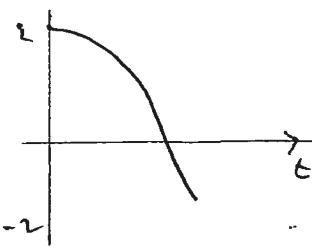
∴ Prob(No English playing each other) $= \frac{24}{105} = \frac{8}{35}$
∴ Prob (At least one final with 2E) $= 1 - 8/35 = \underline{\underline{\frac{27}{35}}}$</td>
<td>1

2</td>
<td>This was complicated by the wording.
If '4' was answered, no less than 1 mark could be awarded.

If the final answer was correct, no less than 2 could be given (Many got extra factor of 4! in both parts).

If complementary probabilities were used, at least ½ was awarded.</td>
</tr>
<tr>
<td>b) i) This is SHM with $n^2 = 1/4$
∴ $n = \frac{1}{2}$ $(n > 0)$
∴ Period $= \frac{2\pi}{n} = 4\pi \doteqdot 12.56...$
∴ Time between successive highs is $\underline{\underline{12.6h}}$ (2D)

ii) 
Let soln be $x = -2\cos\frac{t}{2}$ as shown.
at $t = 0$, $x = -2$
at $t = 1$, $x = -1.76$
∴ rise $\underline{0.24\text{ m}}$.

iii) 
Could use above but use $x = 2\cos\frac{t}{2} \Rightarrow v = \dot{x} = \sin\frac{t}{2}$
at $t = 2$ $v = -\sin 1$
$\doteqdot -0.84$
∴ Tide <u>falling</u> at 0.84 m/h.</td>
<td>1
1

1

1

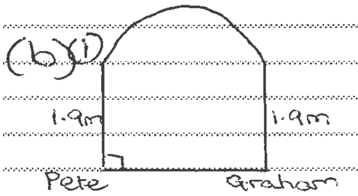
1

1</td>
<td>

Too many people had calculator in degrees mode - they should have smelt a problem.

Could use same equation as (ii) with $t = 2\pi + 2$.</td>
</tr>
</table>

Term 2 2010 **MATHEMATICS Extension 1 : Question** 6

Suggested Solutions	Marks	Marker's Comments
6(a) Let $3\cos 5t + 4\sin 5t = R\cos(5t - \alpha)$ $= R[\cos 5t\cos\alpha + \sin 5t\sin\alpha]$ $\therefore\ 3 = R\cos\alpha \ \ldots (1)$ $4 = R\sin\alpha \ \ldots (2)$ Square (1) + (2) and add $\therefore R = 5$ as $\sin^2\alpha + \cos^2\alpha = 1$ $\therefore$ Max displacement = 5 units	1 1	Only showing $R = \sqrt{3^2+4^2} = 5$ gets a max. of $1\frac{1}{2}$ marks
OR Let $3\cos 5t + 4\sin 5t = R\sin(5t + \alpha)$ $3 = R\sin\alpha \ \ldots (1)$ $4 = R\cos\alpha \ \ldots (2)$ Square (1) + (2) and add $R = 5m$ as $\sin^2\alpha + \cos^2\alpha = 1$ $\therefore$ Max displacement = 5 units	1 1	
(b)(i)  $\ddot{x} = 0$ $\quad \ddot{y} = -10$ $\dot{x} = V\cos\alpha$ $\quad \dot{y} = -10t + V\sin\alpha$ $x = Vt\cos\alpha$ $\quad y = -5t^2 + Vt\sin\alpha + 1.9$ Now $t = 2$, $x = 50$ $\quad$ When $t = 2$, $y = 1.9$ sub into $x = Vt\cos\alpha$ $\quad$ Sub into $y = -5t^2 + Vt\sin\alpha + 1.9$ $50 = 2V\cos\alpha$ $\quad 1.9 = -20 + 2V\sin\alpha + 1.9$ $25 = V\cos\alpha \ \ldots (1)$ $\quad 10 = V\sin\alpha \ \ldots (2)$ (2) ÷ (1) $\frac{V\sin\alpha}{V\cos\alpha} = \frac{10}{25}$ $\therefore \tan\alpha = \frac{2}{5}$ $\angle\alpha = 21°48'$ (nearest minute) ... (3) OR $\alpha = 0.3805^c$ (4dp) Sub (3) in (1) $\frac{25}{\cos 21°48'} = V$ $\therefore V \doteqdot 26.92$ ... (2dp) Exact answer $V = 5\sqrt{29}$ $\therefore$ Velocity $\doteqdot 26.92$ m/s $\quad$ velocity = $5\sqrt{29}$ m/s	 1 1 1	$\frac{1}{2}$ mark for $25 = V\cos\alpha$ $\frac{1}{2}$ mark for $10 = V\sin\alpha$ Leaving answer as $\tan^{-1}\frac{2}{5} = \alpha°$ lost $\frac{1}{2}$ mark
OR $[(1)]^2 + [(2)]^2$ $625 + 100 = V^2\cos^2\alpha + V^2\sin^2\alpha$ $V^2 = 725$ $V = \sqrt{725}$ $\therefore V = 5\sqrt{29}$ ... as $V > 0$ $\therefore$ speed is $5\sqrt{29}$ m/s.	 1	No penalty for $V^2 = 725$, $V = \sqrt{725}$ m/s and not discounting $-\sqrt{725}$ as the question required a speed.

Term 2 2010 **MATHEMATICS Extension 1 : Question** 6

Suggested Solutions	Marks	Marker's Comments
6(b)(ii) For max height $\dot{y}=0$		$\frac{1}{2}$ mark awarded for $t=\frac{V\sin\alpha}{10}$
$-10t+V\sin\alpha=0$		
$-10t+10=0$ as $V\sin\alpha=10$		
$t=1\text{sec}$	1	
at $t=1\text{sec}$		
$y=-5\times1+10\times1+1.9$		
$=10-5+1.9$		
$\therefore y=6.9$		
$\therefore$ Max. height occurs at 6.9m	1	
(iii) $V=5\sqrt{29}$ and $V\sin\alpha=10$, $y=1.9$		$\alpha=45°$ for max range scored $\frac{1}{2}$m
Sub into $y=-5t^2+Vt\sin\alpha+1.9$		
$1.9=-5t^2+Vt\sin\alpha+1.9$	1	
$5t^2=Vt\sin\alpha$		
$5t=5\sqrt{29}\sin\alpha$		
$t=\sqrt{29}\sin\alpha$		
Now $x=Vt\cos\alpha$		
$x=5\sqrt{29}\times\sqrt{29}\sin\alpha\cos\alpha$		
$=145\sin\alpha\cos\alpha$		
$x=\frac{145}{2}\sin2\alpha$	$\frac{1}{2}$	
Max. occurs when $\sin2\alpha=1$		
$\alpha=45°$	$\frac{1}{2}$	
$\therefore R=\frac{145}{2}$		
Graham should stand 72.5m from Pete	1	
OR (iii) Max distance is achieved when $\alpha=45°$	1	
For Graham to catch the ball $y=1.9$m		
Sub into $y=-5t^2+Vt\sin\alpha+1.9$		
$1.9=-5t^2+\frac{Vt}{\sqrt{2}}+1.9$		
$t\left(5t-\frac{V}{\sqrt{2}}\right)=0$		
$\therefore t=0$ or $\frac{V}{\sqrt{2}}=5t$		
$t=\frac{V}{5\sqrt{2}}$		
$=\frac{5\sqrt{29}}{5\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$		
$t=\frac{\sqrt{58}}{2}$	1	
Sub into $x=Vt\cos\alpha$		
$x=5\sqrt{29}\times\frac{\sqrt{58}}{2}\times\frac{1}{\sqrt{2}}$		
$=\frac{5\sqrt{1682}}{2\sqrt{2}}$		
$x=\frac{5}{2}\sqrt{841}=72.5$ (1dp)		
Max distance $=72.5$m	1	

Question 7

(a) $x = -y$ at G.

$$x = -\left(-\tfrac{1}{2}gt^2 + vt\sin\alpha\right)$$

$$x = \tfrac{1}{2}gt^2 - vt\sin\alpha \quad (1/2)$$

$$t.v\cos\alpha = \tfrac{1}{2}gt^2 - vt\sin\alpha \quad (1/2)$$

$$\therefore vt(\cos\alpha + \sin\alpha) = \tfrac{g}{2}t^2$$

$$t \neq 0 \ (1/2). \quad v(\cos\alpha + \sin\alpha) = \tfrac{gt}{2}$$

$$(1/2) \quad \therefore t = \frac{2v(\cos\alpha + \sin\alpha)}{g}$$

(ii) sub $t = \dfrac{2v(\cos\alpha + \sin\alpha)}{g}$ into 'x'

ie $x = vt\cos\alpha$

$$(1/2) \quad = \frac{v.2v(\cos\alpha + \sin\alpha)\cos\alpha}{g}$$

$$(1/2) \quad = \frac{2v^2}{g}(\cos^2\alpha + \sin\alpha.\cos\alpha)$$

$$= \frac{v^2}{g}(2\cos^2\alpha + 2\sin\alpha\cos\alpha)$$

$$(\tfrac{1}{2}) \quad = \frac{v^2}{g}(\sin 2\alpha + \cos 2\alpha + 1)$$

as $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$\cos 2\alpha = 2\cos^2\alpha - 1$
and $\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\quad (1/2)$

$$\therefore OF = \frac{v^2}{g}(\sin 2\alpha + \cos 2\alpha + 1)$$

(iii) At A, $y = 0$

$$\therefore 0 = -\tfrac{1}{2}gt^2 + vt\sin\alpha$$

$$\therefore 0 = t\left(-\tfrac{g}{2} + v\sin\alpha\right)$$

$$t \neq 0 \quad \text{or} \quad t = \frac{2v\sin\alpha}{g} \quad (1/2)$$

distance OA is:

$$x = v\cos\alpha\left(\frac{2v\sin\alpha}{g}\right)$$

$$= \frac{2v^2\sin\alpha\cos\alpha}{g}$$

$$= \frac{v^2\sin 2\alpha}{g} \quad (1/2)$$

Given $OF = \tfrac{4}{3}OA$ (data)

$$\frac{v^2}{g}(\sin 2\alpha + \cos 2\alpha + 1) = \frac{4}{3}.\frac{v^2}{g}.\sin 2\alpha \quad (\tfrac{1}{2})$$

$$\sin 2\alpha + \cos 2\alpha + 1 = \frac{4\sin 2\alpha}{3}$$

$$3\sin 2\alpha + 3\cos 2\alpha + 3 = 4\sin 2\alpha$$

$$3\cos 2\alpha + 3 = \sin 2\alpha \quad (\tfrac{1}{2})$$

$$3(2\cos^2\alpha - 1) + 3 = \sin 2\alpha$$

$$6\cos^2\alpha - 3 + 3 = 2\sin\alpha\cos\alpha$$

$$6\cos^2\alpha - 2\sin\alpha\cos\alpha = 0$$

$$2\cos\alpha(3\cos\alpha - \sin\alpha) = 0 \quad (\tfrac{1}{2})$$

$\therefore \cos\alpha = 0$
$\alpha = 90^\circ$
but $0^\circ < \alpha < 90^\circ$
$\therefore \alpha \neq 90^\circ$ $\quad (\tfrac{1}{2})$

$3\cos\alpha = \sin\alpha$
$3 = \tan\alpha$ $\quad (\tfrac{1}{2})$

$$\therefore \alpha = 71^\circ 34'$$

$$\alpha = 72^\circ \quad (1/2)$$

(nearest degree)